ORF526: Probability Theory

Fall 2023

Instructor:	Liza (Elizaveta) Rebrova
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Class:	Mon and Wed, 11:00 am – 12:20 pm Friend 008
Office hours:	Wed 1:15 pm - 3:15 pm Sherrerd 324
	(from September 20, different time for the first 2 weeks of classes!)
Website:	Canvas https://princeton.instructure.com/courses/11550

Course description

This is a graduate introduction to probability theory with a focus on stochastic processes. Topics include: an introduction to mathematical (measure-theoretic) probability theory. Law of large numbers, central limit theorem, conditioning, filtrations and stopping times. Martingales and martingale convergence. Time permitting, the topics will also include introduction to Markov processes, Poisson processes, and Brownian motion.

The course is designed for PhD students whose ultimate research will involve rigorous mathematical probability. It is a core course for first year PhD students in ORFE and it is also taken by students in several other areas, such as Mathematics, Applied & Computational Mathematics, Computer Science, Economics, Electrical and Computer Engineering, and more.

Textbooks: Your lecture notes are crucial to keep track of the covered material. However, there are many great texts that cover first year graduate probability, with varying focus and scope. Suggested materials include

- A. Dembo, Lecture notes (for a similar course at Stanford), 2021. [online]
- S. Chatterjee, Graduate probability lecture notes (another version of the same Stanford course), 2018. [online]
- E. Çınlar, Probability and Stochastics, 2011. [online]
- R. Durrett, Probability: Theory and Examples (5th Edition), 2019. [online]

Prerequisites:

- Undergraduate probability
- Being comfortable with proof-based mathematics

AI(s):

- **Shambhavi:**
  - **email** ss3472@princeton.edu
  - **office hours** Thu 1.30 - 3.30 pm, Sherrerd 002

Grading

Your grade consists of four parts:

- **Homeworks:** 40% of the grade; 6 problem sets, due **9pm EST** on the days see in the schedule below (no extensions, one lowest score is dropped)
- **Midterm:** 25% of the grade, in class **October 25th** (tentatively but with high probability)
- **Final exam:** 35% of the grade, take-home timed for several hours

All the grades will be released via Gradescope. Any grading complaints and requests for re-grading must be submitted within one week of receiving your score. Grading complaints not initiated within this period of time will not be considered.

(Tentative) homework schedule

1. Homework 1: due Tuesday, September 19
2. Homework 2: due Friday, September 29
3. Homework 3: due Friday, October 13
4. Homework 4: due Friday, November 10
5. Homework 5: due Friday, December 1
6. Homework 6: due Monday December 11

Homework and collaboration policy

- It is always due 9PM.
- No late homework will be accepted.
- Your lowest homework score will be dropped.
- You should attempt solving problems by yourself first. The remaining problems can be discussed with the other students, at the office hours, searched in books and online, **but** in this case (a) you should include the names of your collaborators, books/resources on the first page of the write-up, and (b) do not copy solutions verbatim and always write your individual solution. You do not have to list office hours your attended but a good practice is to mention a particular office hour if it helped you significantly with some problem. None of these collaboration disclosures affect your grades! It is good and encouraged to collaborate and use external resourses for the homework.

- Please be considerate of the grader and write solutions neatly. AIs have the right to not grade unreadable solutions.
- All submissions are done on Gradescope.

Email policy

- Please use email mostly for emergencies and administrative or personal matters.
- For questions about the material and general interest questions, consider coming to office hours and posting to the course Ed Discussion page rather than emailing. We will refer you to the office hours and Ed for the open-ended email questions that were not previously discussed elsewhere.
- However, it makes sense to email with follow-up questions after the office hour discussion rather then wait. Please address and CC' whoever you initiated the conversation with before.
- **You should include "ORF 526", CC' me and the AIs to your emails**, unless the matter is sensitive and you would like to share it strictly personally.

Participation

This is an in-person class by current University guidelines. There will be no zoom, it is the student's responsibility to make up the missed classes material. The aim is to have lectures as interactive as possible, please participate and ask questions in class and after class!

Tentative course plan

- Probability spaces and measures; random variables and distributions
- Independence and product measures
- Integration and expectations, basic inequalities
- Convergence of random variables
- The asymptotics: Laws of large numbers
- Weak convergence and Central Limit Theorem
- Conditional expectation and filtrations
- Martingales, stopping times, Optimal Stopping Theorem
- Martingale convergence
- Intro to Markov chains and ergodic theory
- Intro to Poisson processes
- Intro to Brownian motion